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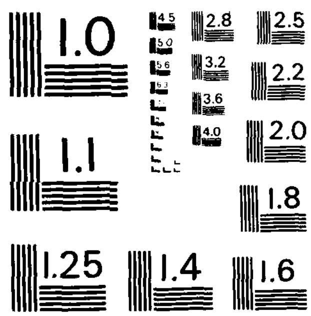

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A channel that introduces intersymbol interference and white gaussian noise is shared by K asynchrouous users that modulate linearly a set of preassigned signal waveforms. In this report an algorithm that implements symbol-by-symbol detection upon observation of an entire data record is derived The algorithm involves nonlinear operations and the forward and backward processing of the data with lineaı complexity in the length of the observation record. The computational requirements of the prepared scheme are compared favorably with a single user Viterbi-like minimum BER algorithm recently proposed by Hayes et. al.

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ON FIXED-INTERVAL MINIMUM SYMBOL ERROR PROBABILITY DETECTION

Consider a data communications model in which the observed process is

$$r(t) = S(t,\underline{b}) + n(t) \quad -MT \leq t < (M+2)T \tag{1}$$

where $n(t)$ is zero-mean white Gaussian noise (spectral height of $N_0/2$), T is the symbol interval duration and $S(t,\underline{b})$ is a linearly modulated K-user information-bearing signal which can be expressed as

$$S(t,\underline{b}) = \sum_{i=-M}^{M} \sum_{k=1}^{K} b_k(i)\ s_k(t - iT - \tau_k) \tag{2}$$

where

$$\underline{b} = \{b(i) = [b_1(i),\ \ldots,\ b_K(i)]^T,\ i = -M,\ \ldots,\ M\};$$

and $b_k(i) \in A$, $s_k(t)$ (assumed to be zero outside a finite interval of positive real line) and $\tau_k \in [0,T)$ are the i^{th} bit, the signal waveform and the delay (modulo T with respect to an arbitrary reference) respectively of the k^{th} user. It is easy to see that the usual models for asynchronous multiple-access communications [1] and for PAM subject to finite-length intersymbol interference [2] are encompassed by the above model.

In this report, we present a forward-backward algorithm that offers MAP symbol-by-symbol detection upon observation of the whole received process, i.e., it solves for

$$b_k(i) \in \arg\max_{a \in A} P[b_k(i) = a \mid r(t),\ -MT \leq t < (M+2)T] \tag{3}$$

$$k = 1,\ \ldots,\ K$$
$$i = -M,\ \ldots,\ M.$$

or equivalently

$$b_k(i) \in \arg\max_{a \in A} \sum_{\substack{\underline{b} \\ \text{s.t. } b_n(i)=a}} \exp[\frac{1}{N_o}\,\Omega(\underline{b})] \tag{4}$$

where

$$\Omega(\underline{b}) = 2\sum_{i=-M}^{M} b^T(i)y(i) - \sum_{i=-M}^{M}\sum_{j=-M}^{M} b^T(i)H(i-j)b(j) \;, \tag{5}$$

and $y(i) = [y_1(i), \ldots, y_K(i)]^T$ is the set of outputs of the K matched filters corresponding to the i^{th} symbol, i.e.,

$$y_k(i) = \int_{\tau_k+iT}^{\tau_k+(i+1)T} r(t)\; s_k(t - iT - \tau_k)\; dt, \tag{6}$$

and the K x K correlation matrices H(i) are defined by

$$H_{kj}(i) = \int_0^T s_k(t)\; s_j(t + iT + \tau_k - \tau_j)\; dt. \tag{7}$$

Because of the assumed finite length of the signals $s_k(t)$, there exists an integer m such that

$$H(i) = \underline{0} \text{ for } |i| > m. \tag{8}$$

The set $S = A^{Kxm}$ will be referred to as the state-space, and the subsets of admissible preceding and next states for a given $\sigma \in S$ are denoted by[1]

$$P(\sigma) = \{x \in S, \text{ s.t. } x^{i+1} = \sigma^i,\; i = 1, \ldots, m-1\}$$

$$N(\sigma) = \{x \in S, \text{ s.t. } x^i = \sigma^{i+1},\; i = 1, \ldots, m-1\}.$$

[1] σ^i denotes the i^{th} column of the matrix σ.

If $\sigma \in S$ and $z \in A^K$ then define the scalar,

$$V(\sigma,z) = \exp[-\frac{1}{N_0} z^T(H(0)z + 2\sum_{j=1}^{m} H(m-j+1)\sigma^j)].$$

Equipped with the above definitions, we can state our solution to (4):

<u>Algorithm</u>

$$\arg\max_{a \in A} \sum_{\substack{\underline{b} \\ \text{s.t. } b_k(i)=a}} \exp[\frac{1}{N_0}\Omega(\underline{b})] =$$

$$\arg\max_{a \in A} \sum_{\substack{\sigma \in S \\ \text{s.t. } \sigma_k^m=a}} F_i(\sigma)\, B_i(\sigma) \tag{9}$$

where $F_k(\cdot)$ and $B_k(\cdot)$ are defined through the forward and backward recursions:

$$F_k(\sigma) = \exp[\frac{2}{N_0} y^T(k)\sigma^m] \sum_{x \in P(\sigma)} F_{k-1}(x)\, V(x,\sigma^m) \tag{10}$$

$$B_{k-1}(\sigma) = \sum_{x \in N(\sigma)} B_k(x)\, V(\sigma,x^m)\, \exp[\frac{2}{N_0} y^T(k)x^m] \tag{11}$$

and the initial conditions:

$$F_k(\sigma) = \exp[\frac{1}{N_0}(2\sum_{i=m-k+1}^{m} y^T(i)\sigma^i - \sum_{i=m-k+1}^{m}\sum_{j=m-k+1}^{m} \sigma^{iT}H(i-j)\sigma^j], \quad k \le m \tag{12a}$$

$$B_M = 1. \tag{12b}$$

<u>Proof</u>

First, organizing the terms in the sum of (4), we have that

$$\sum_{\substack{\underline{b} \\ \text{s.t. } b_k(i)=a}} \exp[\frac{1}{N_0}\Omega(\underline{b})] =$$

$$\sum_{\substack{\sigma \in S \\ \text{s.t. } \sigma_k^m = a}} \quad \sum_{\underline{b} \in W^{-M,M}(\sigma,i)} \exp[\frac{1}{N_0}\ \Omega(\underline{b})], \tag{13}$$

where we have used the notation

$$W^{i,j}(\sigma,k) = \{[b(i),\ \ldots,\ b(j)] \text{ s.t. } b(n) \in A^K \text{ for } n = i,\ldots,j$$
$$\text{and } [b(k-m+1)\ \ \ldots\ \ \ b(k)] = \sigma\},$$

i.e., the set of all symbol subsequences, whose components k-m+1, ..., k coincide with a given state $\sigma \in S$.

Using the fact that $H(i) = H^T(-i)$ (see (7)), it is straightforward to check that

$$\sum_{i=-M}^{M} \sum_{j=-M}^{M} b^T(i)\ H(i-j)\ b(j) = \sum_{i=-M}^{M} b^T(i)[H(0)\ b(i) + 2 \sum_{j=-M}^{i-1} H(i-j)\ b(j)],$$

hence we have

$$\sum_{\underline{b} \in W^{-M,M}(\sigma,i)} \exp[\frac{1}{N_0}\ \Omega(\underline{b})] =$$

$$\sum_{\underline{b} \in W^{-M,M}(\sigma,i)} \exp[\frac{2}{N_0} \sum_{k=-M}^{M} b^T(k)(y(k) - \frac{1}{2}\ H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)\ b(j))]$$

$$\sum_{W^{-M,i}(\sigma,i)} \exp[\frac{2}{N_0} \sum_{k=-M}^{i} b^T(k)(y(k) - \frac{1}{2}\ H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)\ b(j))] \cdot$$

$$\sum_{W^{i+1,M}(\sigma,i)} \exp[\frac{2}{N_0} \sum_{k=i+1}^{M} b^T(k)(y(k) - \frac{1}{2}\ H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)\ b(j))] \tag{14}$$

where the last equality follows from property (8). It remains to show that

both terms in the right hand side of (14) correspond to $F_i(\sigma)$ and $B_i(\sigma)$ respectively, as defined by (10) - (12). To that end, we can write

$$\sum_{W^{-M,i}(\sigma,i)} \exp[\frac{2}{N_0} \sum_{k=-M}^{i} b^T(k)(y(k) - \frac{1}{2} H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)\ b(j))] =$$

$$\sum_{x \in P(\sigma)} \sum_{W^{-M,i-1}(x,i-1)} \exp[\frac{2}{N_0} \sigma^{mT}(y(i) - \frac{1}{2} H(0)\ \sigma^m + \sum_{j=1}^{m} H(m-j+1)x^j)] \cdot$$

$$\exp[\frac{2}{N_0} \sum_{k=-M}^{i-1} b^T(k)(y(k) - \frac{1}{2} H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)b(j))]$$

and the recursive expression (10) follows (for $m < k$).

Analogously,

$$\sum_{W^{i,M}(\sigma,i-1)} \exp[\frac{2}{N_0} \sum_{k=i}^{M} b^T(k)(y(k) - \frac{1}{2} H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)b(j))]$$

$$= \sum_{x \in N(\sigma)} \sum_{W^{i+1,M}(x,i)} \exp[\frac{2}{N_0} x^m(y(i) - \frac{1}{2} H(0)\ x^m + \sum_{j=1}^{m} H(m-j+1)\sigma^j)] \cdot$$

$$\exp[\frac{2}{N_0} \sum_{k=i+1}^{M} b^T(k)(y(k) - \frac{1}{2} H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)b(j))] .$$

Hence, comparing with (11) and (12b) we can identify

$$B_i(\sigma) = \sum_{W^{i+1,M}(\sigma,i)} \exp[\frac{2}{N_0} \sum_{k=i+1}^{M} b^T(k)(y(k) - \frac{1}{2} H(0)b(k) + \sum_{j=-M}^{k-1} H(k-j)b(j))]$$

and the claim is proved.

The forward-backward structure of the above fixed-interval symbol-by-symbol optimum detection algorithm resembles the well-known solution for fixed-interval smoothing (e.g. [3]), the main difference being the independence of the *forward* and *backward recursions*. In fact, the existence of such

algorithms for Markovian decision problems can be traced back to the work by Chang and Hancock [4], and is concisely explained in Forney [5, App.]; see also [6] for a recent application of this idea.

Of particular interest here is the work by Hayes et al. [7], in which they propose an algorithm for solving the minimum symbol error probability problem for our same model (in the special case of 1 user). In this algorithm, the likelihood of every possible value of each symbol is computed by deleting all the trellis states that are not congruent with such value and then running a forward recursion similar to (10). Noticing that at every step the recursion is common for future symbols, the number of computations can be halved. A comparison between the computational complexities of the algorithm by Hayes et al. and the one proposed here seems in order.

The quantities $\exp[\frac{2}{N_0}y^T(k)\cdot z]$, and $V(\sigma,z)$ with $\sigma \in S$ and $z \in A^K$, are common to both algorithms; the first can be computed efficiently digitally or analogically for every input vector and for every symbol combination and the set of second quantities can be precomputed and stored since it does not depend on the data. In any case, the time and space costs incurred in the above computations should be added to those that follow.

Employing the notation, $L = |A|^K$, every step of the forward and backward recursions entails per state: $L + 1$ and $2L$ multiplications, respectively, and $L - 1$ additions, i.e., $(3L + 1)L^m$ multiplications and $(2L - 2)L^m$ additions. The final product of the forward and backward quantities requires one multiplication per state, i.e., L^m multiplications per step, and the computation of the likelihood for each symbol value requires $L^m/|A| - 1$ additions, i.e., $L^m - |A|$ per step.

If the algorithm by Hayes et al. is implemented with a fixed delay of M_2, then every step of the recursion therein is analogous to our forward recursion and requires L − 1 additions and L + 1 multiplications, per state. However, the state trellis is different for each symbol, and the number of additions and multiplications per state must be multiplied by $(1 + M_2L)$. Finally, every possible symbol value requires $L^m - 1$ additions in order to obtain its likelihood. ([7, step 3, p. 155]). The final step of both algorithms is to choose the largest of $|A|$ quantities per symbol.

Regarding global storage requirements, our algorithm needs to store the entire sequences of values of $F_i(\sigma)$ and/or $B_i(\sigma)$, i.e., $2ML^m$ quantities. On the other hand, the algorithm in [7] requires the storage of $L^m(1 + M_2L)$ values. Summarizing, both algorithms have the following complexities:

	HAYES et al.	FORWARD-BACKWARD
ADDITIONS (per symbol)	$L^{m+1}(M_2L - M_2 + 1) - 1$	$L^m(2L - 1) - \|A\|$
MULTIPLICATIONS (per symbol)	$L^m(1 + M_2L)(L + 1)$	$L^m(3L + 2)$
STORAGE	$L^m(1 + M_2L)$	$L^m(2M)$

The key parameter in the comparison of both algorithms is L; if L is large (as is the case in multi-access communications with more than 3 or 4 users) then the forward-backward algorithm offers computational savings of the order of LM_2; if L is small (e.g., binary single-user communications) then the computational effort of the algorithm in [7] is roughly that of the forward-backward algorithm times M_2. Note incidentally that since the forward and backward recursions are independent, there exists, obviously, the possibility of implementing them in parallel.

With respect to the storage requirements, the comparison depends on the relative values of the fixed delay M_2 and M, the fixed length of the data record processed in batch. In general, the choice of M for the fixed interval algorithm will be dictated by the storage capabilities, the allowable decision delay, and the performance degradation due to the partitioning of the data record. It is not unreasonable to expect that comparable levels of degradation will be attained by algorithms whose respective fixed delay M_2 and (half of) fixed length M coincide. Notice that minimum symbol error probability detection can only be justified as an alternative to optimum sequence detection (via Viterbi algorithm) in low signal-to-noise ratio situations since otherwise the optimum decisions according to both criteria do not differ appreciably. The relevance of this point stems from the fact that the delay or data record length required for achieving a given degree of performance degradation increase as the signal-to-noise ratio decreases.

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